Section 4.2 Applications of Extrema (Minimum Homework: 1, 3, 5)

Each problem in section 4.2 is a "geometrical" word problem.

Step 1: Read through the problem to understand what is given, and what needs to be found.
Step 2: Draw a diagram to model the information.
Step 3: Write down any geometrical formula needed to solve the problem.
Step 4: Try to answer each question asked in the problem. I tried to write the problems in such a way that solving the parts of the problem will lead you to the solution.

Let us try to complete problem 4 from section 4.2.

4) A campground owner has 4000 meters of fencing. He wants to enclose a rectangular field bordering a river, with no fencing needed along the river, and let W represent the width of the field and L represent the length of the field. Find the dimensions that maximize the enclosed area.

Make W be the side of the fence that is perpendicular to the river so that two widths and one length will need to be constructed.

Step 1: Read through the problem to understand what is given, and what needs to be found.

- Given – 4000 meters of fencing
- Need to find Length of rectangle
- Need to find the Width of the rectangle
- Goal is to enclose. the maximum area

Step 2: Draw a diagram to model the information.

River – NO FENCE

W = Width

W = Width

4000 meters of fencing will be used to build 1 length and two widths.

Length = L

Step 3: Write down any geometrical formula needed to solve the problem.

Need to know Perimeter formula for the sides of fence that are being constructed.

$$L + 2W = P \quad (P\ is\ the\ amount\ of\ fence\ used\ in\ meters)$$

Need to know an Area formula for a rectangle:

$$A = LW \quad (A = Area\ of\ the\ rectangle\ in\ square\ feet.)$$

Step 4: Try to answer each question asked in the problem. I tried to write the problems in such a way that solving the parts of the problem will lead you to the solution.

a) Write an equation for the length of the field

I know $L + 2W = P$

The $P$ is just the amount of available fencing ($P = 4000$)

$L + 2W = 4000$

The instructions are worded in such a way, that I should solve this equation for $L$.

Part a answer: $L = -2W + 4000$

b) Write an equation for the area of the field.

This is the generic area formula for the rectangle drawn on the last page.

$A = \ LW$

I need to replace the $L\ in\ the\ equation\ with\ L = -2W + 4000$

$A = (-2W + 4000)W$

$A \ = \ -2W^2 + 4000W$

Part b answer: $A \ = \ -2W^2 + 4000W$

c) Find the domain of the area equation that was created in part b.
(This domain will be of the form: $\# \leq W \leq \#$)

This is just a logic question. I want to know the bounds on the Width. I know the width must be at least 0 meters, as it the width cannot be negative.

Also the width cannot be more than 2000 meters, as we must build two widths and there is only 4000 meters of fencing available.

Part c answer: $0 \leq W \leq 2000$

d) Find the value of $W$ leading to the maximum area

This is just an absolute maximum problem. I will create a table that has 0 and 2000 in the x-column based on part c. I will add any appropriate first derivative critical numbers.

$A' = -4W + 4000$

$-4W + 4000 = 0$

$-4W = -4000$

$W = 1000$

| Possible Width | Resulting area |
|---|---|
| $0$ | $A = -2(0)^2 + 4000(0) = 0$ |
| $1000$ | $A = -2(1000)^2 + 4000(1000) = 2{,}000{,}000$ |
| $2000$ | $A = -2(2000)^2 + 4000(2000) = 0$ |

The absolute maximum area is $2{,}000{,}000\ square\ meters$ which happens when $Width = 1{,}000\ meters$

Part d answer: $Width = 1{,}000\ meters$.

e) Find the value of L leading to the maximum area

Use the formula: $L = -2W + 4000 \quad with \ W = 1000$

$L = -2(1000) + 4000 = 2000$

Part e answer: $Length = 2000\, meters$

f) Find the maximum area.

This is already done. It is the y-column in the table.

Part f answer: Maximum area: $2{,}000{,}000\ square\ meters$

Let us now try problem 8 from this section.

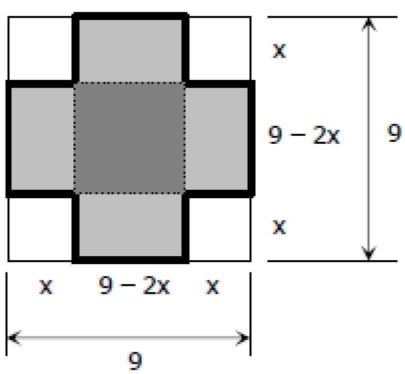
8) An open box is to be made by cutting a square corner of a 9-inch by 9-inch piece of metal then folding up the sides. What size square should be cut from each corner to maximize volume?

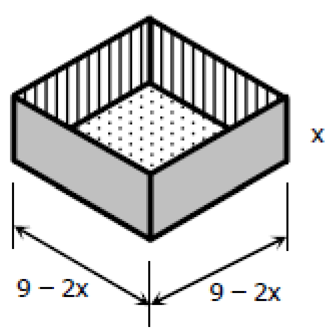
a) Sketch a diagram that models the problem.
b) Write an equation for the volume of the box.
c) Find the domain of the volume equation created in part b.
(This domain will be of the form: $\# \leq x \leq \#$)
d) Find the value of $x$ that makes the volume the largest.
e) Find the maximum volume.

Step 1: Read through the problem to understand what is given, and what needs to be found.

Given a 9-inch by 9-inch square piece of metal.
Need to determine the size of square cuts in each corner.
Goal is to create a box with largest volume.

Step 2: Draw a diagram to model the information.



Step 3: Write down any geometrical formula needed to solve the problem.

Need the volume formula for the box being constructed.

$V = lwh$

$(V = volume\ measured\ in\ cubic\ inches)$

$l =$

$length\ of\ one\ of\ the\ sides\ along\ the\ bottom\ of\ the\ box\ measured\ in\ inches$

$l = 9 - 2x$

$w =$

$length\ of\ the\ other\ side\ along\ the\ bottom\ of\ the\ box\ measured\ in\ inches$

$w = 9 - 2x$

$h = height\ of\ the\ box$ measured in inches

$h = x$

Step 4: Try to answer each question asked in the problem. I tried to write the problems in such a way that solving the parts of the problem will lead you to the solution.

a) Sketch a diagram that models the problem.

See previous page.

b) Write an equation for the volume of the box.

$v = lwh$
$v = (9 - 2x)(9 - 2x)x$
$v = (81 - 18x - 18x + 4x^2)x$
$v = (4x^2 - 36x + 81)x$
Part b answer: $v = 4x^3 - 36x^2 + 81x$

c) Find the domain of the volume equation created in part b.
(This domain will be of the form: $\# \leq x \leq \#$)

The cut must be at least 0 inches, as this amount cannot be negative. The cut must be less than 4.5 inches, as two cuts are being made and they cannot exceed the length of the object.

Part c answer: $0 \leq x \leq 4.5$

d) Find the value of $x$ that makes the volume the largest.

This is just an absolute maximum problem. I will create a table that has 0 and 9 in the x-column based on part c. I will add any appropriate first derivative critical numbers.

$v' = 12x^2 - 72x + 81$
$3(4x^2 - 24x + 27) = 0$
can be solved by factoring or the quadratic formula. I will factor, you can use the quadratic formula if you desire.

$(2x - 9)(3x - 2) = 0$

$2x - 9 = 0$ $\quad\quad$ $3x - 2 = 0$
$2x = 9$ $\quad\quad$ $3x = 2$

$x = \frac{9}{2} = 4.5$ $\quad\quad$ $x = \frac{3}{2} = 1.5$

| Possible value for x (I think decimals are easier) | Resulting Volume |
|---|---|
| 0 | $v = 4(0)^3 - 36(0)^2 + 81(0) = 0$ |
| 4.5 | $v = 4(4.5)^3 - 36(4.5)^2 + 81(4.5) = 0$ |
| 1.5 | $v = 4(1.5)^3 - 36(1.5)^2 + 81(1.5) = 54$ |

This last row of the table gives this information: The absolute maximum volume of the box is 54 $in^3$ which occurs when the cut is 1.5 $inches$

Part d answer: 1.5 $inches$

e) Find the maximum volume.

Part e answer: 54 $cubic\ inches$

1)  A campground owner has 1000 meters of fencing.  He wants to enclose a rectangular field with the fence that he has.  Let W represent the width of the field and L represent the length of the field.  Find the dimensions that maximize the enclosed area.

a)  Write an equation for the length of the field.

b)  Write an equation for the area of the fenced in field.

c)  Find the domain of the area equation that was created in part b.  (This domain will be of the form: $\# \leq W \leq \#$)

d)  Find the value of w leading to the maximum area

e)  Find the value of L leading to the maximum area

f)  Find the maximum area.

2) A campground owner has 5000 meters of fencing. He wants to enclose a rectangular field with the fence that he has. Let W represent the width of the field and L represent the length of the field. Find the dimensions that maximize the enclosed area.

a) Write an equation for the length of the field.

b) Write an equation for the area of the fenced in field.

c) Find the domain of the area equation that was created in part b. (This domain will be of the form: $\# \leq W \leq \#$)

d) Find the value of w leading to the maximum area

e) Find the value of L leading to the maximum area

f) Find the maximum area.

3)  A campground owner has 1000 meters of fencing.  He wants to enclose a rectangular field bordering a river, with no fencing needed along the river, and let W represent the width of the field  and L represent the length of the field.  Find the dimensions that maximize the enclosed area.

Make W be the side of the fence that is perpendicular to the river so that two widths and one length will need to be constructed.

a)  Write an equation for the length of the field

b)  Write an equation for the area of the field.

c)  Find the domain of the area equation that was created in part b. (This domain will be of the form: $\# \leq W \leq \#$)

d)  Find the value of w leading to the maximum area

e)  Find the value of L leading to the maximum area

f)  Find the maximum area.

4)  A campground owner has 4000 meters of fencing.  He wants to enclose a rectangular field bordering a river, with no fencing needed along the river, and let W represent the width of the field  and L represent the length of the field.  Find the dimensions that maximize the enclosed area.

Make W be the side of the fence that is perpendicular to the river so that two widths and one length will need to be constructed.

a)  Write an equation for the length of the field

b)  Write an equation for the area of the field.

c)  Find the domain of the area equation that was created in part b. (This domain will be of the form:  $\# \leq W \leq \#$)

d)  Find the value of w leading to the maximum area

e)  Find the value of L leading to the maximum area

f)  Find the maximum area.

5) An open box with a square base is to be made from a square piece of cardboard 10 inches on a side by cutting out a square ( x inches by x inches) from each corner and turning up the sides.

a) Sketch a diagram that models the problem.
b) Write an equation for the volume of the box.
c) Find the domain of the volume equation created in part b.
(This domain will be of the form: $\# \leq x \leq \#$)
d) Find the value of x that makes the volume the largest.
e) Find the maximum volume.

6) An open box with a square base is to be made from a square piece of cardboard 12 inches on a side by cutting out a square ( x inches by x inches) from each corner and turning up the sides.

a) Sketch a diagram that models the problem.
b) Write an equation for the volume of the box.
c) Find the domain of the volume equation created in part b.
(This domain will be of the form: $\# \leq x \leq \#$)
d) Find the value of x that makes the volume the largest.
e) Find the maximum volume.

7) An open box is to be made by cutting a square corner of a 20 inch by 20-inch piece of metal then folding up the sides. What size square should be cut from each corner to maximize volume?

a) Sketch a diagram that models the problem.
b) Write an equation for the volume of the box.
c) Find the domain of the volume equation created in part b.
(This domain will be of the form: $\# \leq x \leq \#$)
d) Find the value of x that makes the volume the largest.
e) Find the maximum volume.

8)  An open box is to be made by cutting a square corner of a 9-inch by 9-inch piece of metal then folding up the sides.  What size square should be cut from each corner to maximize volume?

a)  Sketch a diagram that models the problem.

b)  Write an equation for the volume of the box.

c)  Find the domain of the volume equation created in part b.
(This domain will be of the form: $\# \leq x \leq \#$)

d)  Find the value of $x$ that makes the volume the largest.

e)  Find the maximum volume.